

NAG Toolbox for MATLAB

f08te

1 Purpose

f08te reduces a real symmetric-definite generalized eigenproblem $Az = \lambda Bz$, $ABz = \lambda z$ or $BAz = \lambda z$ to the standard form $Cy = \lambda y$, where A is a real symmetric matrix and B has been factorized by f07gd, using packed storage.

2 Syntax

```
[ap, info] = f08te(itype, uplo, n, ap, bp)
```

3 Description

To reduce the real symmetric-definite generalized eigenproblem $Az = \lambda Bz$, $ABz = \lambda z$ or $BAz = \lambda z$ to the standard form $Cy = \lambda y$ using packed storage, f08te must be preceded by a call to f07gd which computes the Cholesky factorization of B ; B must be positive-definite.

The different problem types are specified by the parameter **itype**, as indicated in the table below. The table shows how C is computed by the function, and also how the eigenvectors z of the original problem can be recovered from the eigenvectors of the standard form.

itype	Problem	uplo	B	C	z
1	$Az = \lambda Bz$	'U' 'L'	$U^T U$ LL^T	$U^{-T} A U^{-1}$ $L^{-1} A L^{-T}$	$U^{-1} y$ $L^{-T} y$
2	$ABz = \lambda z$	'U' 'L'	$U^T U$ LL^T	$U A U^T$ $L^T A L$	$U^{-1} y$ $L^{-T} y$
3	$BAz = \lambda z$	'U' 'L'	$U^T U$ LL^T	$U A U^T$ $L^T A L$	$U^T y$ $L y$

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **itype** – int32 scalar

Indicates how the standard form is computed.

itype = 1

if **uplo** = 'U', $C = U^{-T} A U^{-1}$;

if **uplo** = 'L', $C = L^{-1} A L^{-T}$.

itype = 2 or 3

if **uplo** = 'U', $C = UAU^T$;
 if **uplo** = 'L', $C = L^TAL$.

Constraint: **itype** = 1, 2 or 3.

2: **uplo** – string

Indicates whether the upper or lower triangular part of A is stored and how B has been factorized.

uplo = 'U'

The upper triangular part of A is stored and $B = U^T U$.

uplo = 'L'

The lower triangular part of A is stored and $B = LL^T$.

Constraint: **uplo** = 'U' or 'L'.

3: **n** – int32 scalar

n , the order of the matrices A and B .

Constraint: $n \geq 0$.

4: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n symmetric matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$)
 for $i \leq j$;
 if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in
ap($i + (2n - j)(j - 1)/2$) for $i \geq j$.

5: **bp**(*) – double array

Note: the dimension of the array **bp** must be at least $\max(1, n \times (n + 1)/2)$.

The Cholesky factor of B as specified by **uplo** and returned by f07gd.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**(*) – double array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The upper or lower triangle of **ap** contains the corresponding upper or lower triangle of C as specified by **itype** and **uplo**, using the same packed storage format as described above.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **itype**, 2: **uplo**, 3: **n**, 4: **ap**, 5: **bp**, 6: **info**.

7 Accuracy

Forming the reduced matrix C is a stable procedure. However it involves implicit multiplication by B^{-1} if (**itype** = 1) or B (if **itype** = 2 or 3). When f08te is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion. See the document for f08sa for further details.

8 Further Comments

The total number of floating-point operations is approximately n^3 .

The complex analogue of this function is f08ts.

9 Example

```
itype = int32(1);
uplo = 'L';
n = int32(4);
ap = [0.24;
      0.39;
      0.42;
      -0.16;
      -0.11;
      0.79;
      0.63;
      -0.25;
      0.48;
      -0.03];
bp = [2.039607805437114;
      -1.529705854077835;
      0.2745625891934577;
      -0.04902903378454601;
      1.640121946685673;
      -0.2499814119483738;
      0.6188564222624378;
      0.7887488055748053;
      0.6442661302310234;
      0.61606333757807];
[apOut, info] = f08te(itype, uplo, n, ap, bp)

apOut =
    0.0577
    0.1704
    0.2950
   -0.6024
    0.2268
    0.8667
   -0.6159
   -0.0500
    0.3972
   -1.6875
info =
    0
```

